Restoring numbers, we have

$$4z^4-2562z^3+10885z^2+3200z-171600=0 \ldots\ldots\ldots\ldots\ldots(4).$$

Solving this equation by *Horner's Method*, we find $z=4$. $\therefore \sqrt{b^2-z^2}=3$.

$\therefore$ From (1) or (2), $y=100$.

If in (3) we let $L=100$, $B=50$, and $b=3$ and solve the equation again for z, we find $z=2.750413+=TI$. $\therefore IW=1.5248$. $\therefore WR=109.4494693746751+$ feet, the required length of the parallelopiped.

Had we solved equation (2) for z and substituted its value in (1), we would have obtained an equation which would give the length of the rectangle $IMLK$, but it would require a great deal of work to free the equation of radicals. We shall now obtain such an equation, or formula.

Let $AB=L$, $BH=B$, $\theta=\angle AIM$, and $x=IM$. Then $AI=x\cos\theta$, $AM=x\sin\theta$, $IB=b\sin\theta$, $BK=b\cos\theta$.

$$\therefore \text{Area of } ABHE=2[\tfrac{1}{2}AI\times AM+\tfrac{1}{2}BI\times BK]+IM\times IK$$

$$=(x^2+b^2)\cos\theta\sin\theta=ab \ldots\ldots\ldots\ldots\ldots(1).$$

Also $$x\cos\theta+b\sin\theta=L \ldots\ldots\ldots\ldots\ldots(2),$$

$$x\sin\theta+b\cos\theta=B \ldots\ldots\ldots\ldots\ldots(3).$$

Squaring (2) and (3) and adding the results, we have

$$x^2+c^2+4cx\sin\theta\cos\theta=L^2+B^2 \ldots\ldots\ldots\ldots\ldots(4).$$

Equating $\sin\theta\cos\theta$ in (1) and (4), we have, after an easy reduction,

$$x^4-(L^2+B^2+2b^2)x^2+4LBbx-b^2(L^2+B^2-b^2)=0 \ldots\ldots\ldots(5),$$

an equation which gives the length of the longest rectangle of given width which can be diagonally inscribed in a given rectangle.

[NOTE.—It is but justice to Mr. Bell to say that he was obliged to protest long and vigorously before he received a proper hearing to his claim that the published solution of Dr. Matz and Mr. Burleson is wrong. It was simply a case of that injustice commonly done to men when we believe them to be wrong and refuse to examine their claims. This problem was proposed a few years ago in the *School Visitor*, and at that time we solved the problem though we did not try to obtain the numerical result. When Dr. Matz and Mr. Burleson sent in their solution, it seemed to us on cursory examination to be obtained on the same plan pursued by us a few years ago. But after Mr. Bell had written to us on several different occasions, we offered to publish his solution that our readers might compare the results. But before doing so, we examined the published solution in the May No. Vol. II and found that it was wrong. The numerical calculation of $z=WR$ is due to Mr. Bell, as is also the last equation and the method of obtaining it. EDITOR.]

49. Proposed by J. C. WILLIAMS, Rome, New York.

Of all triangles inscribed in a given segment of a circle, with the chord as base, the isosceles is the maximum.

I. Solution by M. A. GRUBER, War Department, Washington, D. C., and A. P. REED, Superintendent of Schools, Clarence, Missouri.

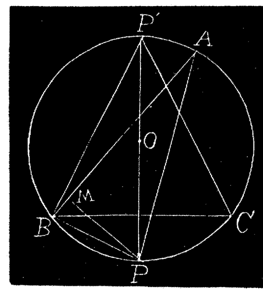
The bases being equal, the maximum triangle is the one having the greatest altitude.

In any segment of a circle, the greatest perpendicular that can be drawn to the chord, is the perpendicular to the middle of the chord. This perpendicular is the altitude of the isosceles triangle.

$\therefore$ The isosceles triangle is the maximum.

II. Solution by J. M. COLAW, A. M., Superintendent of Schools, Monterey, Virginia, and E. KESNER, Boulder, Colorado.

As the segment may be greater or less than a semi-circle, the general proof is for the circle. In the figure it is obvious that the isosceles triangle $P'BC$ is greater than any other triangle ABC, *as its altitude is greater*. Having the given chord as the common base, the *area* depends entirely on the altitude. But the isosceles triangle is a maximum both in *perimeter* and *area*.



Draw PM perpendicular to AB. Then the triangles APM, $P'PB$ are similar, and the diameter $P'P$ is $>AP$; $\therefore$ $P'B$ is $>AM$.

But $2AM=AB+CA$ (*Richardson and Ramsey's Modern Plane Geometry*, *pp*. 24, 131).

$\therefore$ $P'BC$ has the maximum perimeter.

Also solved by *E. L. SHERWOOD* and *G. B. M. ZERR*.

[NOTE.—This problem, with the addition that the isosceles triangle has the maximum perimeter, is Theorem 11, page 131, *Richardson and Ramsey's Modern Plane Geometry*. EDITOR.]

PROBLEMS.

54. Proposed by I. J. SCHWATT, Ph. D., University of Pennsylvania, Philadelphia, Pennsylvania.

Prove geometrically :

If through the center of perspective D of a given triangle ABC and its Brocard triangle $A'B'C'$ be drawn straight lines so as to pass through S_a, S_b and S_c (S_a, S_b, and S_c are the middle points of the sides BC, AC, and AB of the triangle ABC) and if S_aD_1 is made equal to DS_a, S_bD_2 equal to DS_b, and S_cD_3 equal to DS_c then are (1) the figures $D_1O'AO$, $D_2O'BO$ and $D_3O'CO$ parallelograms (O and O' are Brocard's points), (2) the triangles $D_1D_2D_3$ and ABC are equal, and (3) D_1A, D_2B, and D_3C intersect in S, (S is the middle point of OO').

55. Proposed by FREDERICK R. HONEY, Ph. B., New Haven, Connecticut.

Let ab and cd be respectively the major and minor axes of an ellipse, and let α be the angle which a diameter th forms with the major axis ; it is required to find the length of this diameter.